## Introduction

Mafia is a social deduction game created by Dimitry Davidoff in 1986 in which a group of 'town' players try to uncover and eliminate a smaller group of 'mafia' players. Players are slowly removed from the game, either from being voted out or killed by the mafia. The game plays out in several night/day cycles' and ends once either all mafia are eliminated, or the mafia has a voting majority.

Basic Terminology

* Town: The good guys. The uninformed majority. Their goal is to work together during the day to discover which players are secretly the mafia.
* Mafia: The bad guys. The informed majority. Their goal is to eliminate all town players during the night before being discovered.
* Moderator: A player who doesn't belong to either team, instead overseeing the events that take place throughout the game and communicating the results to the players (eg. Saying who the mafia killed, or the result of a vote).
* Day: The part of the game in which all players talk amongst themselves. At the end of each day, one player is eliminated by a majority vote.
* Night: The mafia silently decide amongst themselves on a player to eliminate.

### Basic Special Roles

The town have several (optional) special roles to help them beat the mafia, since without them the mafia have a much larger probability of winning, which we will discuss later. Most special roles have several variations on their rules, which we will go into in a later section.

* Cop (investigative roles): The cop can investigate one player each night and discover whether they are mafia or town. (I strongly recommend using the first alternate rule I describe later on)
* Medic (protective roles): The medic can choose one player each night to save. That player cannot be killed by the mafia that night.
* Vigilante (killing roles): The vigilante can choose a player to kill during the night. They can do this once over the course of the game. (DO NOT use the vigilante if you want to have fun. I will show you why later on)

### Getting Started

The version of the game we will be talking about is the version played in real life, with a group of players and a moderator. It is recommended to play with at least seven players and a moderator. The number of mafia in a given game is usually equal to , where is the number of players. The remaining players are town, with one given the cop role, and one the medic role. This setup is the one I use, and I will assume it is being used for the rest of the paper.

## Calculations

### Total games played

If we want to calculate the total number of games played, we need to simplify the game and define all of its parts. For this first part, let’s assume a simple game that consists of town players and mafia players where (It would be difficult to play with a fraction of a person, or negative players). For now, we won’t include any special roles. We will break the game down into its day/night cycles, starting with night and alternating between the two until the game ends. At the start of the day, the game ends if , as the mafia will have voting majority and can kill off the rest of the town. At the start of the night, the game ends if .

During the day, three things can happen:

1. The town successfully kills a mafia
2. The town kills another town
3. The town doesn’t kill anyone

For the night, let’s assume that the mafia kills a town every time.

After each night, the game progresses to a new day with one fewer town player. After each day, the game progresses to a new night with either one fewer mafia, one fewer town, or the same number of players. Since these same rules can be applied to every day and night until the game ends, we can build recursive functions to describe them.

/\*

Otherwise, we can call our function again for every possible outcome of the night/day cycle and add them together to get the total number of games played. Outcome 1 gives us, output 2 gives and output 3 gives. We always subtract 1 from the number of town since the mafia is killing a town every night. Putting this all together gives us the following function:

If we want to see how many times mafia wins, we only need to make one small adjustment to our function. Instead of returning 1 at the end of the game, we return 1 only if the game ends in a mafia win. If the town wins, we want to return 0 instead.

We can divide this value by the total number of games to estimate the likelihood that the mafia will win a given game. For example:

Looking at this function makes it clear that the town deciding not to kill anyone is always a bad idea in this simplified version of the game. Removing that option from the function lowers the mafia win rate and helps simplify the function a bit.

\*/

### Predicting game outcomes

If we want to be more accurate with our mafia win rate prediction, we need to take the probabilities of certain events into account, as not every outcome has an equal likelihood of occurring. First, let’s build a function that predicts the outcome of a simple game. If the town wins it will return 0, and if the mafia wins it will return 1. We can use the same base cases as our number of mafia wins function.

Next, instead of calling our function again for all possible outcomes, we want to call it once, depending on what happens during the day. If a town dies, we will call and if a mafia dies, we will call. We can predict what happens during the day by choosing a random player, and checking to see whether or not they are mafia. That can be accomplished by the following steps:

1. Choose a random number from 0 to (total number of players)
2. If < then you have chosen a mafia player
3. Otherwise, you have chosen a town player

Based on what player we choose, call the corresponding function.

This function predicts whether or not mafia wins for a single game, so running this function over many games should give a more accurate estimate for mafia win rates. For example, running the function 1,000,000 times results in:

This is quite a high win rate for the mafia; let’s add something to our prediction to help the town. In the game of mafia, there are several special roles that the town can take on, one of them being the detective (or cop).